Spectator Effects in Inclusive Decays of Beauty Hadrons *

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arXiv:hep-lat/9809083v3 2 Oct 1998

We evaluate the matrix elements of the four-quark operators which contribute to the lifetimes of B-mesons and the Λ_b-baryon. We find that the spectator effects are not responsible for the discrepancy between the theoretical prediction and experimental measurement of the ratio of lifetimes $\tau(\Lambda_b)/\tau(B)$.

1. INTRODUCTION

Inclusive decays of heavy hadrons can be studied in the framework of the heavy quark expansion, in which, for example, lifetimes are computed as series in inverse powers of the mass of the b-quark [1]. For an arbitrary hadron H

$$\tau^{-1}(H) = \frac{G_F^2 m_b^5}{192\pi^3} \frac{|V_{cb}|^2}{2m_H} \sum_{i\geq 0} c_i m_b^{-i} \tag{1}$$

where

- c_0 corresponds to the decay of a free-quark and is universal.
- c_1 is zero because the operators of dimension four can be eliminated using the equations of motion.
- c_2 can be estimated and is found to be small.
- c_3 contains a contribution proportional to

  $$\langle H|\ \bar{b}\Gamma q\ \bar{q}\widetilde{\Gamma} b\ |H\rangle \tag{2}$$

  and is therefore the first term in the expansion to which the interaction between the heavy and the light quark(s) contribute. Although this is an $O(m_b^{-3})$ correction, it may be significant since it contains a phase-space enhancement.

The aim of our lattice simulation is to compute c_3 for B-mesons and the Λ_b-baryon, in order to check whether spectator effects contribute significantly to the ratios of lifetimes for which the experimental values are:

$$\frac{\tau(B^-)}{\tau(B^0)} = 1.06 \pm 0.04 \tag{3}$$

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = 0.78 \pm 0.04\ . \tag{4}$$

The discrepancy between the experimental value in eq. (4) and the theoretical prediction of $\tau(\Lambda_b)/\tau(B^0) = 0.98$ (based on the Operator Product Expansion in eq. (1) including terms in the sum up to those of $O(m_b^{-2})$) is a major puzzle. It is therefore particularly important to compute the $O(m_b^{-3})$ spectator contributions to this ratio.

The ratios in eqs. (3) and (4) can be expressed in terms of 6 matrix elements:

$$\frac{\tau(B^-)}{\tau(B^0)} = a_0 + a_1\varepsilon_1 + a_2\varepsilon_2 + a_3 B_1 + a_3 B_2 \tag{5}$$

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = b_0 + b_1\varepsilon_1 + b_2\varepsilon_2 + b_3 L_1 + b_4 L_2 \tag{6}$$

where [2]

$$B_1 \equiv \frac{8}{f_B^2 m_B} \frac{\langle B|\ \bar{b}\gamma^\mu L q\ \bar{q}\gamma_\mu L b\ |B\rangle}{2m_B} \tag{7}$$

$$B_2 \equiv \frac{8}{f_B^2 m_B} \frac{\langle B|\ \bar{b} L q \bar{q} R b\ |B\rangle}{2m_B} \tag{8}$$

$$\varepsilon_1 \equiv \frac{8}{f_B^2 m_B} \frac{\langle B|\ \bar{b}\gamma^\mu L t^a q\ \bar{q}\gamma_\mu L t^a b\ |B\rangle}{2m_B} \tag{9}$$

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[2] In terms of the parameters $\widetilde{B}$ and r introduced in ref. [2]

$$r = -6L_1$$
$$\widetilde{B} = -2L_2/L_1 - 1/3$$

$$\varepsilon_2 \equiv \frac{8}{f_B^2 m_B} \frac{\langle B| \ \bar{b}Lt^a q \ \bar{q}Rt^a b \ |B\rangle}{2m_B} \tag{10}$$

$$L_1 \equiv \frac{8}{f_B^2 m_B} \frac{\langle \Lambda| \ \bar{b}\gamma^\mu Lq \ \bar{q}\gamma_\mu Lb \ |\Lambda\rangle}{2m_\Lambda} \tag{11}$$

$$L_2 \equiv \frac{8}{f_B^2 m_B} \frac{\langle \Lambda| \ \bar{b}\gamma^\mu Lt^a q \ \bar{q}\gamma_\mu Lt^a b \ |\Lambda\rangle}{2m_\Lambda} \tag{12}$$

and the coefficients a_i and b_i are given by

	value		value
a_0	$+1.00$	b_0	$+0.98$
a_1	-0.697	b_1	-0.173
a_2	$+0.195$	b_2	$+0.195$
a_3	$+0.020$	b_3	$+0.030$
a_4	$+0.004$	b_4	-0.252

The values in the table correspond to operators renormalized in a continuum renormalization scheme at the scale $\mu = m_B$. Their matrix elements are obtained from those in the lattice regularization by perturbative matching (see the appendix).

2. B DECAY

The matrix elements $B_1, B_2, \varepsilon_1, \varepsilon_2$ are computed on a $24^3 \times 48$ lattice at $\beta = 6.2$ (corresponding to a lattice spacing $a^{-1} = 2.9(1)$ GeV) using the tree-level improved SW action for three values of $\kappa = 0.14144, 0.14226, 0.14262$ and are then extrapolated to the chiral limit ($\kappa_c = 0.14315$) [3]. We find

$$B_1 = 1.06 \pm 0.08 \tag{13}$$

$$B_2 = 1.01 \pm 0.06 \tag{14}$$

$$\varepsilon_1 = -0.01 \pm 0.03 \tag{15}$$

$$\varepsilon_2 = -0.02 \pm 0.02 \tag{16}$$

which implies that

$$\frac{\tau(B^-)}{\tau(B^0)} = 1.03 \pm 0.02 \pm 0.03 \tag{17}$$

in agreement with the experimental value (3).

3. Λ DECAY

The computation the baryonic matrix elements L_1 and L_2 is a little more difficult. We have performed an exploratory study in which the light quark propagators are computed using a stochastic method [4] based on the relation

$$M_{ij}^{-1} = \int [d\phi](M_{jk}\phi_k)^* \phi_i e^{-\phi_i^*(M^+M)_{ij}\phi_j} \tag{18}$$

(rather then using "extended" propagators).

The matrix elements are computed on a $12^3 \times 24$ lattice at $\beta = 5.7$ (corresponding to a lattice spacing $a^{-1} = 1.10(1)$ GeV) for two values of κ. We therefore do not attempt an extrapolation to the chiral limit ($\kappa_c = 0.14351$) but present results seperately for each value of κ. We find:

$$L_1 = \begin{cases} -0.30 \pm 0.03 & (\kappa = 0.13843) \\ -0.22 \pm 0.03 & (\kappa = 0.14077) \end{cases} \tag{19}$$

$$L_2 = \begin{cases} 0.23 \pm 0.02 & (\kappa = 0.13843) \\ 0.17 \pm 0.02 & (\kappa = 0.14077) \ , \end{cases} \tag{20}$$

which implies that (neglecting the systematic error due to the chiral extrapolation)

$$\frac{\tau(\Lambda_b)}{\tau(B^0)} = \begin{cases} 0.91 \pm 0.01 & (\kappa = 0.13843) \\ 0.93 \pm 0.01 & (\kappa = 0.14077) \ . \end{cases} \tag{21}$$

These results imply that spectator effects are not sufficiently large to explain the discrepancy between the theoretical prediction and experimental result in eq. (4).

4. CONCLUDING REMARKS

We find that the matrix elements of the 4-quark operators (13-16) satisfy the vacuum saturation hypothesis remarkably well. A similar feature is true for the $\Delta B = 2$ operators which contribute to B–$\bar{B}$ mixing. We do not have a good understanding yet of this phenomenon.

The results for the mesonic matrix elements lead to a prediction for the ratio of the lifetimes of the charged and neutral mesons which is in agreement with the experimental result in eq. (3). Our study of the baryonic matrix elements indicates that spectator effects are not sufficiently large to explain the experimental ratio of lifetimes in eq. (4). This discrepancy between the theoretical prediction and the experimental measurement remains an important problem to solve. We do stress, however, that our calculations are exploratory, and a more precise simulation is necessary, in particular to allow for a reliable extrapolation to the chiral limit.

Table 1
Lattice perturbative coefficients and operators

p_i	P_i	q_i	Q_i
$-\frac{4}{3}\log(\lambda^2a^2)+8.46$	$\overline{b}\Gamma q\overline{q}\widetilde{\Gamma}b$	$+1$	$\overline{b}\Gamma q\overline{b}\widetilde{\Gamma}q$
$-\log(\lambda^2a^2)+6.19$	$\overline{b}t^a\Gamma t^a q\overline{q}\widetilde{\Gamma}b+\overline{b}\Gamma q\overline{q}t^a\widetilde{\Gamma}t^a b$	$+1$	$\overline{b}t^a\Gamma t^a q\overline{b}\widetilde{\Gamma}q+\overline{b}\Gamma q\overline{b}t^a\widetilde{\Gamma}t^a q$
-6.89	$\overline{b}t^a\gamma^0\Gamma\gamma^0 t^a q\overline{q}\widetilde{\Gamma}b+\overline{b}\Gamma q\overline{q}t^a\gamma^0\widetilde{\Gamma}\gamma^0 t^a b$	$+1$	$\overline{b}t^a\gamma^0\Gamma\gamma^0 t^a q\overline{b}\widetilde{\Gamma}q+\overline{b}\Gamma q\overline{b}t^a\gamma^0\widetilde{\Gamma}\gamma^0 t^a q$
$2\log(\lambda^2a^2)-4.53$	$\overline{b}t^a\Gamma q\overline{q}\widetilde{\Gamma}t^a b$	-1	$\overline{b}t^a\Gamma q\overline{b}t^a\widetilde{\Gamma}q$
$\log(\lambda^2a^2)-6.19$	$\overline{b}\Gamma t^a q\overline{q}\widetilde{\Gamma}t^a b+\overline{b}t^a\Gamma q\overline{q}t^a\widetilde{\Gamma}b$	-1	$\overline{b}\Gamma t^a q\overline{b}t^a\widetilde{\Gamma}q+\overline{b}t^a\Gamma q\overline{b}\widetilde{\Gamma}t^a q$
-6.89	$\overline{b}\Gamma\gamma^0 t^a q\overline{q}\widetilde{\Gamma}\gamma^0 t^a b+\overline{b}t^a\gamma^0\Gamma q\overline{q}t^a\gamma^0\widetilde{\Gamma}b$	$+1$	$\overline{b}\Gamma\gamma^0 t^a q\overline{b}t^a\gamma^0\widetilde{\Gamma}q+\overline{b}t^a\gamma^0\Gamma q\overline{b}\widetilde{\Gamma}\gamma^0 t^a q$
$-\log(\lambda^2a^2)+5.12$	$\overline{b}\Gamma t^a q\overline{q}\gamma^0\widetilde{\Gamma}b$	-1	$\overline{b}\Gamma t^a q\overline{b}\widetilde{\Gamma}t^a q$
$-\frac{1}{4}\log(\lambda^2a^2)+1.05$	$\overline{b}\Gamma\sigma^{\mu\nu}t^a q\overline{q}t^a\sigma^{\nu\mu}\widetilde{\Gamma}b$	-1	$\overline{b}\Gamma\sigma^{\mu\nu}t^a q\overline{b}\widetilde{\Gamma}\sigma^{\mu\nu}t^a q$
-2.43	$\overline{b}\Gamma\gamma^\mu t^a q\overline{q}t^a\gamma^\mu\widetilde{\Gamma}b$	$+1$	$\overline{b}\Gamma\gamma^\mu t^a q\overline{b}\widetilde{\Gamma}\gamma^\mu t^a q$

The analytic expression for p_i can be found in [3].

ACKNOWLEDGEMENTS

It is a pleasure to thank Chris Michael, Hartmut Wittig, Jonathan Flynn, Luigi Del Debbio, Giulia De Divitiis, Vicente Gimenez and Carlotta Pittori for many helpful discussions. This work was supported by PPARC grants GR/L29927, GR/L56329 and GR/I55066.

APPENDIX: 1-loop lattice perturbative corrections to 4-quark operators

The most difficult component of the evaluation of the 1-loop perturbative matching between four-quark lattice operators and those renormalized in a continuum scheme is the perturbative expansion on lattice. In this appendix we present the corresponding results for generic operators P_0 and Q_0 (whose matrix elements contribute to lifetimes and B–$\bar{B}$ mixing respectively):

$$P_0 \equiv \overline{b}\Gamma q\, \overline{q}\widetilde{\Gamma}b, \qquad (\Delta B=0) \tag{22}$$

$$Q_0 \equiv \overline{b}\Gamma q\, \overline{b}\widetilde{\Gamma}q, \qquad (\Delta B=2)\ . \tag{23}$$

$\Gamma\otimes\widetilde{\Gamma}$ represents an arbitrary spinor and color tensor. These operators mix under renormalization with other 4-quark operators, listed in table 1 [3]:

$$P_0^{1\,\mathrm{loop}} = P_0+\frac{\alpha_s(a^{-1})}{4\pi}\sum_i p_i P_i \tag{24}$$

$$Q_0^{1\,\mathrm{loop}} = Q_0+\frac{\alpha_s(a^{-1})}{4\pi}\sum_i p_i q_i Q_i\ . \tag{25}$$

[3]These results were also obtained independently in ref. [6].

The Feynman rules correspond to the tree-level SW-improved action for massless light quarks and with a small gluon-mass (λ) as the infrared regulator. The dependence on λ, of course, cancels when the corresponding continuum calculation is combined with the lattice one.

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